Глава IV. Методы исследования математических моделей

4. Асимптотические методы

Метод малого параметра. Сингулярные возмущения

23 22 Так как в (17) μ=0

$$(25)\ \Rightarrow\ \mathrm{F}_0(\mathrm{t}) = \overset{23}{=}\mathrm{F}\Big|_{\mu=0} \overset{22}{=} \mathrm{f}(\mathrm{y}_0(\mathrm{t}),\mathrm{t}) = 0 \qquad (26)$$

$$\mathfrak{F}_0(\tau) \overset{23}{=} \mathfrak{F}\Big|_{\mu=0} \overset{22}{=} f(y_0(0)+\Pi_0(\tau),0) - \underbrace{f(y_0(0),0)}_{\overset{26}{=0}} = f(y_0(0)+\Pi_0(\tau),0) \qquad (30)$$

$$(21)\ \Rightarrow\ y(0,\mu) = y_0(0) + \mu y_1(0) + \ldots + \Pi_0(0) + \mu\Pi_1(0) + \ldots =$$

$$\overset{18}{=} y^0 = y_0^0 + \mu y_1^0 + \ldots \qquad (31)$$

$$(31)\ \Rightarrow\ \Pi_0(0) = y_0^0 - y_0(0) \qquad (32)$$

$$\begin{array}{l} (28)\ \Rightarrow \\ (32)\ \Rightarrow \end{array} \begin{cases} \dfrac{d\Pi_0}{d\tau} = \mathfrak{F}_0(\tau) \overset{30}{=} f(y_0(0)+\Pi_0(\tau),0), \tau > 0, & (33) \\ \Pi_0(0) = y_0^0 - y_0(0) & (34) \end{cases}$$

$$(27)\ \Rightarrow\ \frac{dy_0}{dt} = F_1(t) \overset{23}{=} \frac{\partial F(t)}{\partial\mu}\Big|_{\mu=0} \overset{22}{=} \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial\mu}\Big|_{\mu=0} = f_y(y_0(t),t)y_1(t) \qquad (35)$$

$$(29)\ \Rightarrow\ \frac{d\Pi_1}{d\tau} = \mathfrak{F}_1(\tau) = \frac{\partial\mathfrak{F}(t)}{\partial\mu}\Big|_{\mu=0} = f_y(y_0(0)+\Pi_0(\tau),0)\frac{\partial y}{\partial\mu}\Big|_{\mu=0} - f_y(y_0(0),0)\frac{\partial y}{\partial\mu}\Big|_{\mu=0} +$$

$$f_t(y_0(0)+\Pi_0(\tau),0)\frac{\partial t}{\partial\mu}\Big|_{\mu=0} - f_t(y_0(0),0)\frac{\partial t}{\partial\mu}\Big|_{\mu=0} =$$

$$= f_y(y_0(0)+\Pi_0(\tau),0)\Big(\frac{\partial y_0(\mu\tau)}{\partial t}\overset{=\tau}{\frac{\partial t}{\partial\mu}} + y_1(\mu) + \mu\frac{\partial y_1(\mu\tau)}{\partial t}\overset{=\tau}{\frac{\partial t}{\partial\mu}} + \ldots +$$

$$+\underline{\Pi_1(\tau)} + 2\mu\Pi_2(\tau)\Big)\Big|_{\mu=0} - f_y(y_0(0),0)\Big(\frac{\partial y_0(\mu\tau)}{\partial t}\overset{=\tau}{\frac{\partial t}{\partial\mu}} + y_1(\mu) + \mu\frac{\partial y_1(\mu\tau)}{\partial t}\overset{=\tau}{\frac{\partial t}{\partial\mu}}\Big)\Big|_{\mu=0} +$$

$$+f_t(y_0(0)+\Pi_0(\tau),0)\frac{\partial t}{\partial\mu}\Big|_{\mu=0}-f_t(y_0(0),0)\frac{\partial t}{\partial\mu}\Big|_{\mu=0}=f_y(y_0(0)+\Pi_0(\tau),0)\Pi_1(\tau)+$$

$$\underbrace{\boxed{+(f_y(y_0(0)+\Pi_0(\tau),0)-f_y(y_0(0),0)(y_0'(0)\tau+y_1(0))+(f_t(y_0(0)+\Pi_0(\tau),0)-f_t(y_0(0),0))\tau}}_{=Q_1}=$$

$$=f_y(y_0(0)+\Pi_0(\tau),0)\Pi_1(\tau)+Q_1 \qquad (36)$$

$$(31)\ \Rightarrow\ y(0,\mu)=y_0(0)+\underline{\underline{\mu y_1(0)}}+\ldots+\Pi_0(0)+\mu\Pi_1(0)+\ldots=$$

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$$=y^0=y_0^0+\underline{\underline{\mu y_1^0}}+\ldots\quad\Rightarrow$$

$$\Pi_1(0)=y_1^{\,0}-y_1(0) \qquad (37)$$

Цепочка решения:

Алгебраическое уравнение:

$$F_0(t) = f(y_0(t), t) = 0 \qquad (26)$$

$$y_0(t)$$

Задача Коши:

$$\begin{cases} \dfrac{d\Pi_0}{d\tau} = f(y_0(0) + \Pi_0(\tau), 0), \tau > 0, & (33) \\ \Pi_0(0) = y_0^0 - y_0(0) & (34) \end{cases}$$

$$\Pi_0(\tau)$$

Алгебраическое уравнение:

$$\frac{dy_0}{dt} = f_y(y_0(t), t) y_1(t) \qquad (35)$$

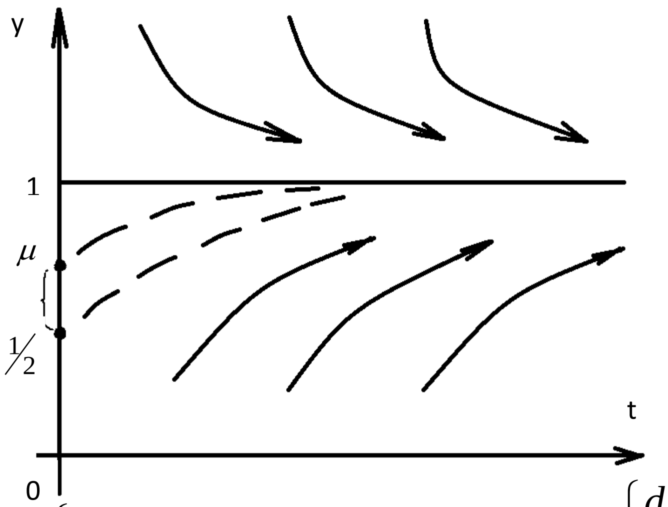
$$y_1(t)$$

Задача Коши:

$$\begin{cases} \dfrac{d\Pi_1}{d\tau} = f_y(y_0(0) + \Pi_0(\tau), 0)\Pi_1(\tau) + Q_1, \tau > 0, & (36) \\ \Pi_1(0) = y_1^0 - y_1(0) \end{cases}$$

$$\Pi_1(\tau)$$

Пример:



$$\begin{cases} \mu \dfrac{dy}{d\tau} = y - y^2, 0 < t \le 1, & (40) \\ y(0) = \dfrac{1}{2} + \mu = y_0^0 + \mu y_1^0 & (41) \end{cases}$$

$(26) \Rightarrow f(y_0(t), t) = 0 \Rightarrow$

$$y_0(t) - y_0^2(t) = 0 \Rightarrow y_0(t) = 1$$

$$\begin{cases} \dfrac{d\Pi_0}{d\tau} = f(y_0(0) + \Pi_0(\tau), 0) \\ \Pi_0(0) = y_0^0 - \underset{=1}{y_0(0)} \end{cases} \Rightarrow \begin{cases} \dfrac{d\Pi_0}{d\tau} = (1 + \Pi_0) - (1 + \Pi_0)^2, \tau > 0, (42) \\ \Pi_0(0) = \dfrac{1}{2} - 1 = -\dfrac{1}{2} \end{cases}$$

$$\boxed{\frac{dy_0}{dt}}^{=0} = f_y(y_0(t), t) y_1(t) \quad (35) \quad \Leftarrow \quad \Pi_0(\tau) = -\frac{1}{1 + e^{\tau}}$$

$$y_1(t) = 0 \Rightarrow y(t, \mu) = \overset{=1}{y_0(t)} + \mu \overset{=0}{y_1(t)} + \Pi_0(\tau) + \mu \Pi_1(\tau) + \ldots =$$

$$= 1 + \Pi_0(\tau) + \underline{\underline{O}}(\mu) = 1 - \frac{1}{1 + e^{\tau}} + \underline{\underline{O}}(\mu) \qquad (44)$$

$$\begin{matrix} (36) \\ (37) \\ (41) \\ (44) \end{matrix} \Rightarrow \begin{cases} \dfrac{d\Pi_1}{d\tau} = (1 - 2(1 - \dfrac{1}{1 + e^{\tau}}))\Pi_1, \tau > 0, \\ f_y = 1 - 2y \\ \Pi_1(0) = y_1^0 - y_1(0) = 1 - 0 = 1 \quad \Rightarrow \end{cases}$$

$$\Pi_1(\tau) = \frac{4e^{\tau}}{(1+e^{\tau})^2} \;\Rightarrow\; y(t,\mu) = \overset{=1}{y_0}(t) + \mu \overset{=0}{y_1}(t) + \Pi_0(\tau) +$$

$$+\mu \Pi_1(\tau) + \underline{\underline{O}}(\mu^2) = 1 - \frac{1}{1+e^{\tau}} + \mu \frac{4e^{\tau}}{(1+e^{\tau})^2} + \underline{\underline{O}}(\mu^2) \quad (48)$$