

**Глава II. Некоторые классические задачи математической физики**  
**§5. Задача о промерзании (задача о фазовом переходе, задача Стефана)**

**Построение решения задачи (1) – (4):**

$$(2) \Rightarrow u_1 = A_1 + B_1 \Phi\left(\frac{x}{2a_1\sqrt{t}}\right); u_2 = A_2 + B_2 \Phi\left(\frac{x}{2a_2\sqrt{t}}\right)$$

$$x=0: A_1 = T_1; \quad t=0: A_2 + B_2 = T$$

$$(3) \Rightarrow A_1 + B_1 \Phi\left(\frac{x}{2a_1\sqrt{t}}\right)_{x=\xi} = A_2 + B_2 \Phi\left(\frac{x}{2a_2\sqrt{t}}\right)_{x=\xi} = 0$$

$$A_1 + B_1 \Phi\left(\frac{x}{2a_1\sqrt{t}}\right)_{x=\xi} = 0 \Rightarrow B_1 = -\frac{T_1}{\Phi\left(\frac{\xi}{2a_1\sqrt{t}}\right)}$$

$$A_2 = T - B_2 = -B_2 \Phi\left(\frac{\xi}{2a_2\sqrt{t}}\right) \Rightarrow$$

$$T = B_2 \left(1 - \Phi\left(\frac{\xi}{2a_2\sqrt{t}}\right)\right) \Rightarrow B_2 = \frac{T}{1 - \Phi\left(\frac{\xi}{2a_2\sqrt{t}}\right)}$$

$$A_2 = T - B_2 = T - \frac{T}{1 - \Phi\left(\frac{\xi}{2a_2\sqrt{t}}\right)} = -\frac{T \Phi\left(\frac{\xi}{2a_2\sqrt{t}}\right)}{1 - \Phi\left(\frac{\xi}{2a_2\sqrt{t}}\right)}$$

$$(4) \Rightarrow \frac{\partial u_1}{\partial x} = -\frac{T_1}{\Phi\left(\frac{\alpha}{2a_1}\right)} \frac{\partial}{\partial x} \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{2a_1\sqrt{t}}} e^{-z^2} dz = -\frac{T_1}{\Phi\left(\frac{\alpha}{2a_1}\right)} \frac{2}{\sqrt{\pi}} \frac{1}{2a_1\sqrt{t}} e^{-\frac{x^2}{4a_1^2 t}}$$

$$\frac{\partial u_2}{\partial x} = -\frac{T}{1 - \Phi\left(\frac{\alpha}{2a_2}\right)} \frac{2}{\sqrt{\pi}} \frac{1}{2a_2\sqrt{t}} e^{-\frac{x^2}{4a_2^2t}}$$

$$-k_1 \frac{T_1}{a_1\sqrt{\pi t} \Phi\left(\frac{\alpha}{2a_1}\right)} e^{-\frac{\xi^2}{4a_1^2t}} - k_2 \frac{T}{a_2\sqrt{\pi t} \Phi\left(1 - \Phi\left(\frac{\alpha}{2a_2}\right)\right)} e^{-\frac{\xi^2}{4a_2^2t}} =$$

$$= \lambda\rho \frac{d}{dt}(\alpha\sqrt{t}) = \frac{\lambda\rho\alpha}{2\sqrt{t}}, \quad \text{где } \alpha = \frac{\xi}{\sqrt{t}} \Rightarrow$$

$$\frac{k_1 T_1 e^{-\frac{\alpha^2}{4a_1^2}}}{a_1 \Phi\left(\frac{\alpha}{2a_1}\right)} + \frac{k_2 T_1 e^{-\frac{\alpha^2}{4a_2^2}}}{a_2 \left(1 - \Phi\left(\frac{\alpha}{2a_2}\right)\right)} = -\lambda\rho\alpha \frac{\sqrt{\pi}}{2}$$

$$\text{При } T=0: B_2 = \frac{T}{1 - \Phi\left(\frac{\alpha}{2a_2}\right)} = 0 \Rightarrow A_2 = T - B_2 = 0:$$

$$A_1 = T_1; B_1 = -\frac{T_1}{\Phi\left(\frac{\alpha}{2a_1}\right)}; A_2 = 0; B_2 = 0$$

$$\frac{k_1 T_1 e^{-\frac{\alpha^2}{4a_1^2}}}{a_1 \Phi\left(\frac{\alpha}{2a_1}\right)} = -\lambda\rho\alpha \frac{\sqrt{\pi}}{2} \quad (10)$$

$$\text{Положим } \beta = \frac{\alpha}{2a_1}, \quad D = \frac{\lambda\rho a_1^2}{k_1 T_1}$$

$$\text{Тогда из (10)} \Rightarrow \frac{1}{\sqrt{\pi}} \frac{e^{-\beta^2}}{\Phi(\beta)} = -D\beta$$